EARTHQUAKE ENGINEERING AND STRUCTURAL DYNAMICS, VOL. 25, 31–46 (1996)

THE N2 METHOD FOR THE SEISMIC DAMAGE ANALYSIS OF RC BUILDINGS

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SUMMARY

A comprehensive, though relatively simple, non-linear method for the seismic damage analysis of reinforced concrete buildings (the N2 method) has been elaborated. The basic features of the method are: the use of two separate mathematical models, application of the response spectrum approach and of the non-linear static analysis, and the choice of a damage model which includes cumulative damage. The method yields results of reasonable accuracy provided that the structure oscillates predominantly in the first mode. Three variants of a seven-storey building have been used as illustrative examples for the application of the method. Four different types of the analysis, with different degrees of sophistication, have been performed in order to estimate the influence of several assumptions and approximations used in the N2 method.

KEY WORDS: inelastic seismic analysis; seismic design; earthquake resistant structures; reinforced concrete buildings; damage evaluation

INTRODUCTION

For the rational aseismic design of new buildings, as well as for the seismic evaluation of existing buildings, a procedure is needed which would, firstly, yield an adequate estimate of demand in terms of structural stiffness, strength, ductility and energy dissipation, and which would, secondly, not be more complicated than necessary, taking into account the uncertainties connected with the input data. The methods applied in building codes are based on the assumption of linear elastic structural behaviour and fail to meet the first requirement. On the other hand, the non-linear dynamic time-history analysis of Multi-Degree-Of-Freedom (MDOF) mathematical models is not practical for everyday design use. Such an analysis requires additional data (time-histories of several ground motions and the hysteretic behaviour of structural members), whereas the results are, due to uncertainties in the input data, not necessarily more reliable.

Recently, several researchers have emphasized the need for changes in the existing seismic design methodology implemented in codes (e.g. References 1–5). Attention has been focused on the explicit incorporation of displacement demand and inelastic response characteristics including cumulative damage, in the design process. In Japan, new design guidelines for reinforced concrete buildings have been developed.[6] These guidelines require a static non-linear analysis as the main analysis procedure, obligatory for buildings from 31 to 60 m high. Adequate performance of the structure has to be ensured for the prescribed limit deformations. According to Eurocode 8,[7] for a very limited number of cases (some types of steel and composite buildings), the design seismic loading depends on the actual strength of the structure that may be determined by non-linear static analysis. This type of analysis, in connection with Eurocode 8, has been discussed in detail by Mazzolani and Piluso.[8] The need for a new seismic design methodology is becoming the focus of several major efforts in the U.S.A. (e.g. SEAOC Vision 2000, ATC-33, ATC-34, BSSC NEHRP Updates).

In this paper a method is described which is intended to achieve a satisfactory balance between required reliability and applicability for everyday design use, and which might contribute to the practical implementation of new trends in seismic design. The basic features of the method are: the use of two separate mathematical models, the application of the response spectrum approach and of the non-linear static (push-over) analysis, and the choice of a damage model which includes cumulative damage. Cumulative damage is considered to be especially important for existing structures which have frequently not been

CCC 0098-8847/96/010031-16

Received 13 October 1994
Revised 18 July 1995

detailed for sustained resistance through many cycles of response into the inelastic range. Applications of the method are, for the time being, restricted to the planar analysis of building structures vibrating predominantly in the fundamental mode. Some problems (e.g. bidirectional input and the influence of coupling of the fundamental translational and torsional modes) have still to be solved before the method can be extended to the analysis of three-dimensional models of buildings. The proposed method can be used for the seismic evaluation of both existing and newly designed buildings. In the latter case, it represents the second step in the design process, the first one being a normal code design procedure, where the same inelastic spectra as in the proposed method can be used. The method will be referred to as the N2 method, where N stands for non-linear, and 2 for two mathematical models.

The results of past research work, performed in recent years at the University of Ljubljana, have been incorporated into the present version of the method. It basically represents a further development of the N2 method proposed in References 9 and 10 and extended in Reference 11. Consistent inelastic response spectra developed in References 12–14, as well as the modified Park–Ang damage model (originally proposed by Park *et al.*[15]), have been used. It should be noted, however, that the suggested procedures used in particular steps of the method can be easily replaced by other available procedures. The same basic idea of using two mathematical models has been applied by several other researchers (e.g. References 16–20). In the study reported here, cumulative damage considerations and general purpose inelastic spectra have been included, and the details of the computational procedure are elaborated.

In the paper, the method is first described. Then it is applied to the analysis of three seven-storey RC building structures. The frame-wall building, tested in Tsukuba, Japan, within the joint U.S.–Japan project, was used as the first test structure. Two variants of the same structure, but without the wall (the strong-columns and weak-beams concept as well as a weak first-storey concept were used in the design), have been analysed in the second and third test example. The results have been compared with the results of planar MDOF time-history analyses, and with the results of combined static MDOF and dynamic Single-Degree-Of-Freedom (SDOF) system analysis.

DESCRIPTION OF THE METHOD

The basic steps of the method are summarized in Figure 1. Here, only some comments are given.

Structural data depend, to some extent, on the mathematical model used for the non-linear static analysis. Structural members are usually modelled as elastic beams with concentrated plasticity (hinges) at their ends. For each hinge, the moment (M)–rotation (Θ) relationship is needed, including the ultimate rotation under monotonic loading (Θ_u), which represents the deformation capacity. A great deal of uncertainty still exists about the estimation of seismic capacity. A procedure which is based on the elementary principles of structural mechanics and includes some semi-empirical terms was developed in New Zealand (e.g. Reference 21). As an alternative, completely empirical formulae are often used. Some closed-form empirical expressions appropriate for RC structural members have been incorporated into the IDARC program.[22] Recently, non-parametric multi-dimensional regression analysis was proposed for the estimation of seismic capacity.[23] Capacity issues will not, however, be discussed in this paper.

The vertical distribution of lateral loads in the push-over analysis corresponds to the distribution of inertia forces due to the assumed displacement shape (see Appendix II).

Various procedures have been proposed in the literature for the determination of an equivalent SDOF system. In this study the formulae given in Appendix II were used. They do not require transformation of the loading (spectra).

Global seismic demand in terms of the maximum displacement and in terms of the parameter γ (related to hysteretic energy) can be determined using consistent spectra recently developed at the University of Ljubljana.[12–14] Demand values depend, to some extent, on hysteretic behaviour and damping. A stiffness-degrading hysteretic model is appropriate for RC structures.

The rotation and rotation ductility demand in different structural members are assumed to be approximately equal to the corresponding demands in the MDOF model at a static top displacement corresponding to the maximum displacement of the equivalent SDOF model. The energy demand is included implicitly in

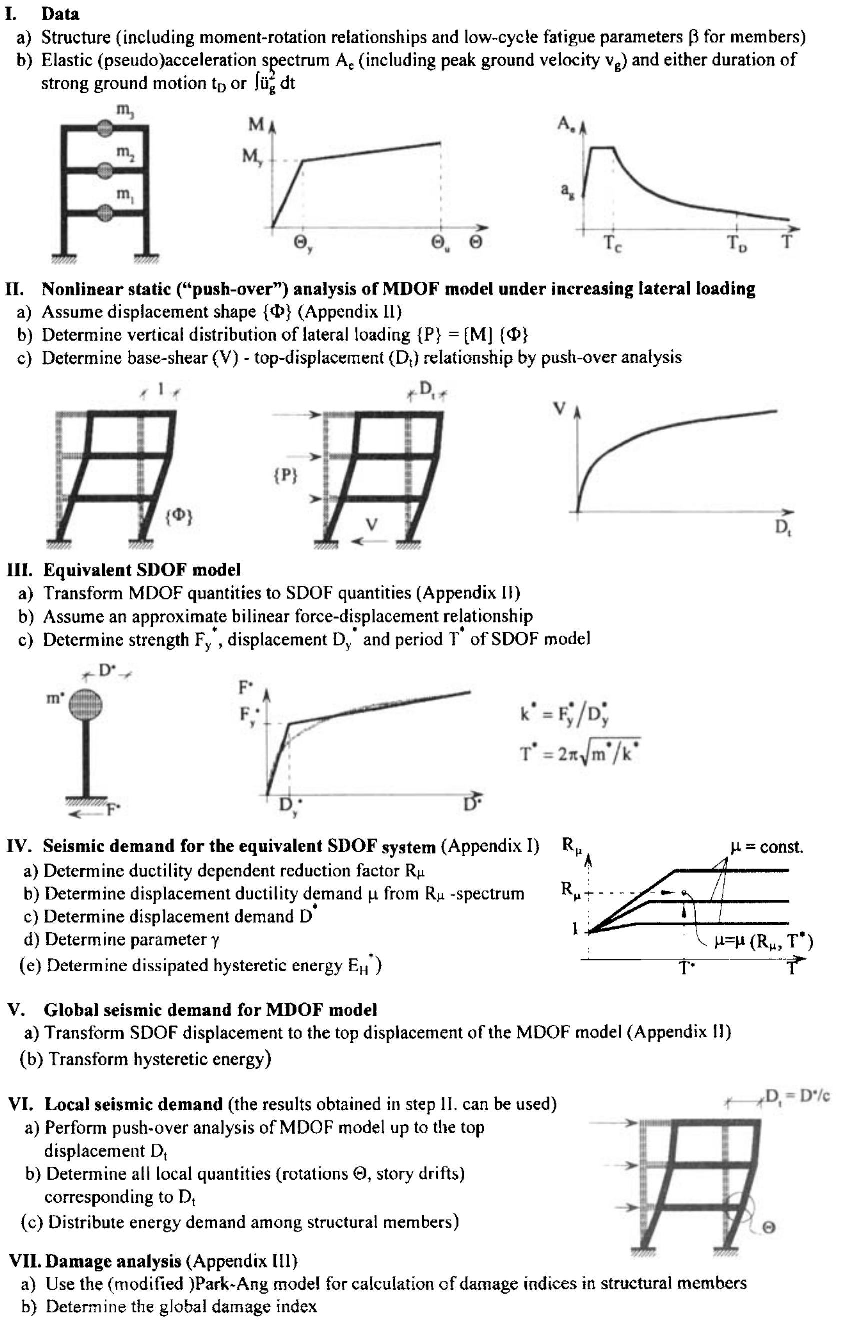

Figure 1. Basic steps of the N2 method

parameter γ (see Appendix I). If, for some reason, the hysteretic energy is needed explicitly, then it can be determined for the equivalent SDOF system (Appendix I), transformed to the MDOF system (Appendix II), and distributed to the members of the MDOF system proportionally to the energy dissipated under monotonically increasing static loading (until maximum displacement is reached).

Knowing the seismic demand and capacity for each structural member, a damage index can be computed for each such member. If needed, a single damage index can be determined for the structure as a whole, based

on the weighted average of damage indices for its structural elements. Among damage models currently available, the Park–Ang model is considered to be the most appropriate for application in the N2 method. A special form of the Park–Ang model was developed which eliminates the need for explicit consideration of hysteretic energy (Appendix III), and thus simplifies the computational procedure. The weights for determining the global damage index were assumed to be proportional to the dissipated energy.

DISCUSSION OF APPROXIMATIONS AND LIMITATIONS

The most essential assumption in the development of the method is a time-independent lateral displacement shape of the structure. The influence of the higher modes cannot, therefore, be taken into account properly. Due to this fact both the absolute values as well as the distribution of all quantities determining seismic demand are influenced. Consequently, the application of the method may be limited mainly to building structures oscillating predominantly in a single (fundamental) mode, even if they are irregular (e.g. weak and/or soft first-storey structures). Some proposals have been made as to how to find approximate procedures for the consideration of higher mode effects (e.g. References 24 and 25). However, with the exception of the problems of shear demand[26–28] and pure flexural wall systems,[29] where promising results have been obtained, so far no generally acceptable solution has yet been proposed.

Discrepancy between the results of dynamic and push-over analysis may also occur in exterior columns and beams of the lower storeys of tall and slender buildings where the effects of the tension in the columns are important[3] or in the gravity load dominated frames where plastic hinges form in beams.[30] In such cases, cumulative inelastic deformation under displacement reversals may be additve and the results of push-over analysis can significantly underestimate the local cumulative plastic rotations.[3,30]

The fundamental questions concerning the applicability of push-over analysis, which is a main part of the N2 method, its feasibility and limitations, have been addressed in a recent paper.[31]

The inelastic response spectra used in the N2 method (Appendix I) depend on the basic structural and ground motion characteristics. Approximately, they represent idealized mean spectra determined by a statistical analysis. Typical coefficients of variation amount to about 0·3 for strength and displacement, and to about 0·6 for energies.[12–14] The spectra correspond to an idealized force–displacement envelope and to an idealized hysteretic behaviour, neither of which fits the actual structural behaviour exactly. A relatively large error can be introduced by using such spectra. In a design or evaluation process, however, this error cannot be avoided due to uncertainties in the expected ground motion characteristics and, to some extent, also in the inelastic structural behaviour.

The computed damage indices depend to a high degree on the seismic capacity of the structural elements. The latter is subject to large uncertainties, and is not discussed in this paper.

NUMERICAL EXAMPLES

Structural and ground motion data

Two seven-storey RC building structures were chosen as test examples. The first building completely corresponds to the RC frame-wall building (Figure 2) tested in Tsukuba within the framework of the joint U.S.–Japan research project. More data on this structure are given elsewhere (e.g. Reference 32). In the second building, the structural wall was omitted. Consequently, the structure consists of three identical frames. The frames were designed according to the strong column–weak beam concept (Model 1). A model corresponding to the weak storey concept (Model 2) has also been studied, as an alternative. The concrete cross-sections of the structural elements are the same in both models. In all columns longitudinal reinforcement of 8 ϕ 22 mm is provided, with the exception of the first storey of Model 2 (4 ϕ 16). All the beams in Model 2 are forced to behave elastically. It should be noted that Model 2 has, in the first storey, a strength discontinuity, but no stiffness discontinuity. Both models have the same initial natural periods.

Three different input accelerograms were chosen in order to include ground motions of different characteristics (especially as regards frequency content and duration). The basic data of the accelerograms are given in

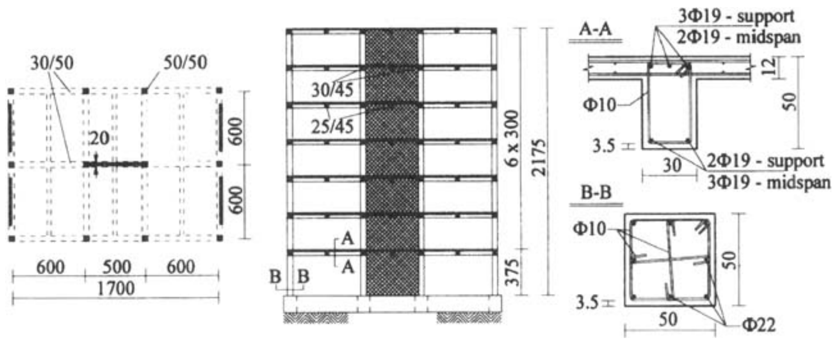

Figure 2. Plan, elevation and typical cross-section of structural members of the frame-wall building

Table I. Basic data on ground motions

Accelerogram	a_g (cm/s^2)	v_g (cm/s)	d_g (cm)	t_D (s)	$\int \ddot{u}_g^2\,dt$ (cm^2/s^3)	Scaling factor: Frame-wall	Scaling factor: Frame
El Centro S00E	342	33·5	10·9	24·0	111000	1·95	1·50
Bar EW	353	52·0	15·1	15·2	174200	0·92	0·72
Llolleo N10E	640	41·1	14·2	34·7	940000	1·60	1·30

Table I (a_g, v_g and d_g are the peak ground accelerations, velocities and displacements of the original ground motions, t_D is the duration of strong ground motion according to Trifunac and Brady,[33] and $\ddot{u}_g$ is the ground acceleration as a function of time). The Bar accelerogram was recorded during the 1979 Montenegro earthquake. All accelerograms were scaled (using a trial and error procedure) in order to obtain a target maximum top displacement of about 1 per cent of the height of the MDOF models in the case of the non-linear dynamic analysis. To be more precise, the maximum top displacements amounted to about 22 and 24 cm for the frame-wall and frame-Model 1 building, respectively. For Model 2, the same scaling of records was used as for Model 1. Two per cent mass-proportional damping, which exaggerates higher mode effects, was chosen. The initial fundamental periods of the structures, based on uncracked cross-sections, were 0·45 and 0·74 s for the frame-wall and frame structure, respectively. Considering the fundamental periods of the structures and the elastic response spectra of the input motions shown in Figure 3, the largest higher mode effects can be expected in the case of the Llolleo input motion.

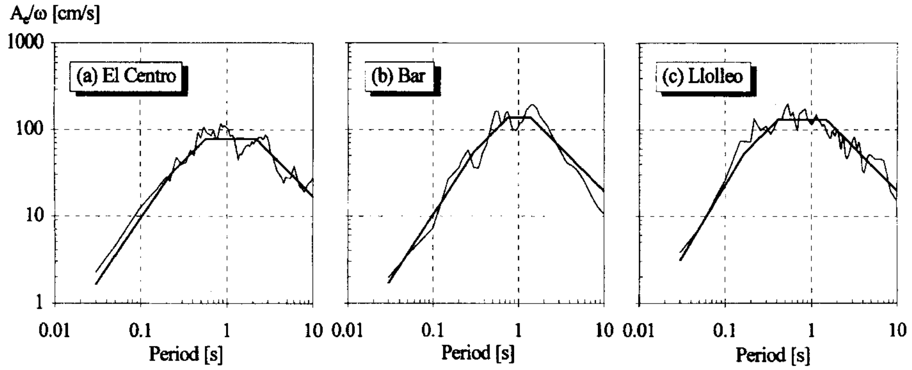

Figure 3. Actual and smoothed elastic pseudovelocity response spectra (2 per cent damping)

All analyses of MDOF systems were performed using the IDARC-L program.[34] This program is based on the IDARC program,[22] which has, however, been subjected to major modifications. *Inter alia*, a multiple-vertical-line-element was implemented for the modelling of structural walls.[35] The MDOF mathematical model was validated by comparing the results of MDOF dynamic analysis with the test results for the frame-wall building obtained in Tsukuba during the PSD3 test. Details of the mathematical modelling and the results of the validation procedure are presented in Reference 34. Here, only some information on the ultimate rotation capacity Θ_u and on the parameter β is given, which control the seismic capacity in the Park–Ang damage model.

Θ_u for beams and columns was estimated by the corrected empirical formula from the IDARC program.[22] The computed values (about 2 and 3 per cent, for columns and beams, respectively) agree well with available experimental results. Based on an empirical formula[34] the ultimate rotation in the structural wall was estimated to be $\Theta_u = 1{\cdot}81$ per cent.

A three-parameter hysteretic model[22] was used for all structural members. The parameter β controls the degradation of reloading stiffness as a function of dissipated energy. It is also a parameter in the Park–Ang damage model. In the original IDARC program an empirical formula is used to determine the parameter β, depending on the characteristics of a structural member. The same β value is used for both the hysteretic and the damage model. In the IDARC-L program these two values are not necessarily the same. It should be noted that the value of β in the hysteretic model has a rather small effect on the global response of the structure in terms of maximum displacements, whereas the influence of β on dissipated hysteretic energy is more important. The value of β in the damage model may significantly influence the damage index. In the case of low β values (e.g. $\beta = 0{\cdot}05$, as suggested by Park *et al.*[36] for reinforced concrete structures), the influence of low-cycle fatigue is small, and maximum displacement (or rotation) governs the seismic behaviour and damage. If the value of β is high, as in the case of the poor seismic design typical for many older existing structures, cumulative damage is usually responsible for failure. In the study reported here, β amounts to 0·15 for all structural members, for both the hysteretic and the damage model. This value is, according to Cosenza *et al.*,[37] an average value for reinforced concrete structures. Using $\beta = 0{\cdot}15$, the contribution of cumulative damage to the damage index may be noticeable. It should be noted, however, that the empirical formula in the original IDARC program yields lower β values for the structural members of the investigated buildings. Further research is needed in order to obtain more reliable β values. However, it does appear to the authors of this paper that the β values obtained by using the original formula proposed by Park *et al.*,[15] as well as those obtained by using the modified Park formula of the original IDARC program,[22] are rather low. Relatively high β values have also been reported in some recent experimental studies (e.g. References 38 and 39).

Types of analysis

In order to validate the proposed N2 method (denoted here as the Level 1 analysis), to evaluate the influence of the assumptions made in different steps, and to demonstrate the method's applicability, several types of analysis were used.

'Exact' results were obtained by non-linear time-history analysis of MDOF systems (Level 4).

In the Level 3 analysis the non-linear static (push-over) analysis was applied. The lateral loading was increased monotonically until the target top displacement D_t, equal to the maximum top displacement from the Level 4 analysis, was reached. The global hysteretic energy demand E_H, determined by the Level 4 analysis, was used. Thus, only one approximation, i.e. fixed displacement shape, was introduced. Displacement shapes according to the proposals in Figure 10 were assumed.

An additional approximation was introduced in the Level 2 analysis, where displacement (D^*) and hysteretic energy (E_H^*) demand were determined from a non-linear time-history analysis of an equivalent SDOF system. Thus, the computed values represent the 'exact' values for a SDOF system. The corresponding D_t and E_H values for the MDOF system are, however, approximations.

The Level 1 analysis (N2 method) includes a third important approximation (in addition to the two approximations involved in the Level 2 analysis), i.e. the determination of D^* and γ from inelastic spectra (Appendix I). Smoothed response spectra of the Newmark–Hall type (Figure 3) were used as input for the

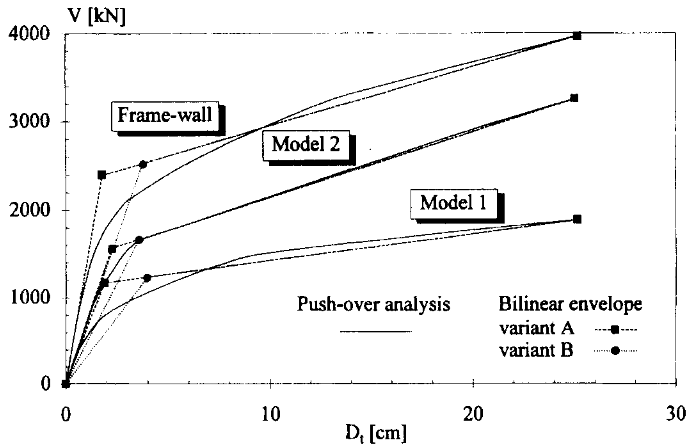

Figure 4. Base shear–top displacement relationships and their idealizations

computation of inelastic spectra. In the Level 2, Level 3 and Level 4 analyses the original Park–Ang damage model, as interpreted in the original IDARC program, was used. In the Level 1 analysis, however, both the original Park–Ang damage model (which explicitly includes E_H) and the modified damage model (where E_H is replaced by γ) were applied.

Two bilinear envelopes of the force–displacement diagrams were used for each structure in order to check the sensitivity of the force–displacement idealizations (Figure 4). In variant A the initial stiffness was based on uncracked sections. In variant B this stiffness was divided by a factor of 2 in the case of the frame-wall and frame-Model 1 structures, and by a factor of 1·5 in the case of the frame-Model 2 structure. These values were chosen arbitrarily, in order to provide an approximate lower bound of the range of elastic stiffness a designer might choose. The elastic periods of the idealized bilinear systems amounted to 0·45, 0·76 and 0·79 s for the frame-wall, frame-Model 1 and frame-Model 2 buildings, respectively (variant A). The corresponding values for variant B amounted to 0·64, 1·07 and 0·97 s. The post- yielding stiffness was the same in both variants (Figure 4). For the frame-wall and frame-Model 1 structures, the ratios of post-yielding to elastic stiffness amounted to 5 and 10 per cent for variants A and B, respectively. In the case of the frame-Model 2 structure, the post-yielding stiffness amounted to 10·65 and 16 per cent of the elastic stiffness for variants A and B, respectively.

Discussion of results

The main results of the analyses of the test structures are shown in Table II and in Figures 5–9. In Table II the global values (top displacement, base shear, dissipated hysteretic energy and global damage index), the damage index in the most critical (first) storey, as well as the damage index in the top storey are shown. The influence of the different assumptions used in the N2 method can be evaluated by comparing the results corresponding to different levels of the analysis. From the results of the Level 1 and Level 2 analyses the effect of using the proposed general-purpose inelastic spectra instead of the time-history analysis of the equivalent SDOF system can be observed. The error introduced by using SDOF system demand for MDOF systems can be estimated by comparing the results obtained by using the Level 2 and Level 3 analyses. The effect of assuming that the distribution of rotation and energy demand throughout the structure is similar to the distribution corresponding to the static behaviour at the maximum top displacement can be observed by comparing the results obtained by the Level 3 and Level 4 analyses.

It can be seen from Table II that, in the majority of cases, the simplified analyses (Levels 1–3) are able to provide reasonably accurate predictions of global seismic demand, as well as of global and storey damage indices. They can clearly detect the weak storey in the frame-Model 2 structure. They can also take into account the higher cumulative damage in the case of long-duration Llolleo ground motion. The major

deviation from the 'exact' (Level 4) results is due to neglecting the contribution of the higher modes to hysteretic energy in the case of Llolleo ground motion (only for the frame-wall and frame-Model 1 buildings). In addition to hysteretic energy, base shear determined using the equivalent SDOF system may be underestimated as well. An additional conclusion which can be drawn from Table II is that the lower initial stiffness of the equivalent SDOF system yields, as a rule, larger displacements.

In Figures 5 and 6 the displacement shapes and storey drifts along the height of the buildings have been plotted for Level 3 (push-over) and Level 4 (dynamic time- history) analyses (in the latter case envelopes are

Table II. Global seismic demand (top displacement D_t, base shear V, dissipated hysteretic energy E_H) and damage indices DM. Damage indices correspond to the original Park–Ang model and to the modified model (values in parentheses). The complexity of the analyses increases from Level 1 (the N2 method) upwards. Variants A and B differ in the assumed initial (elastic) stiffness of the system

Structure	Ground motion	Level of analysis-variant	D_t (cm)	V (kN)	E_H (kN m)	DM Global	DM First storey	DM Top storey
Frame-wall	1·95 El Centro	4	21·9	5010	1451	0·41	0·40	0·55
		3	21·9	3790	1451	0·55	0·43	0·61
		2-A	23·6	3870	1433			
		2-B	25·4	3990	1446			
		1-A	16·8	3410	1454	0·45 (0·48)	0·36 (0·38)	0·51 (0·55)
		1-B	19·9	3620	1223	0·48 (0·52)	0·38 (0·42)	0·54 (0·59)
	0·92 Bar	4	22·0	3910	1033	0·42	0·37	0·48
		3	22·0	3790	1033	0·48	0·38	0·53
		2-A	20·7	3660	1121			
		2-B	24·8	3950	1212			
		1-A	11·9	3080	1040	0·32 (0·33)	0·26 (0·26)	0·37 (0·39)
		1-B	16·0	3360	1024	0·38 (0·42)	0·30 (0·34)	0·43 (0·48)
	1·60 Llolleo	4	21·7	5570	4512	0·58	0·77	0·58
		3	21·7	3780	4512	1·04	0·77	1·19
		2-A	23·0	3830	2539			
		2-B	28·0	4160	2554			
		1-A	25·9	4020	5970	1·34 (1·31)	0·97 (1·04)	1·54 (1·42)
		1-B	27·8	4150	4936	1·22 (1·30)	0·91 (1·04)	1·38 (1·40)
Frame-Model 1	1·50 El Centro	4	24·4	1550	698	0·50	0·56	0·11
		3	24·4	1870	698	0·72	0·85	0·02
		2-A	19·6	1630	576			
		2-B	21·8	1780	573			
		1-A	15·3	1580	574	0·46 (0·61)	0·57 (0·67)	0·00 (0·00)
		1-B	21·6	1780	642	0·65 (0·77)	0·77 (0·89)	0·02 (0·00)
	0·72 Bar	4	24·3	1930	475	0·54	0·73	0·05
		3	24·3	1870	475	0·65	0·77	0·02
		2-A	21·7	1780	423			
		2-B	26·2	1920	439			
		1-A	15·3	1580	615	0·47 (0·62)	0·59 (0·68)	0·00 (0·00)
		1-B	18·5	1680	440	0·53 (0·64)	0·63 (0·73)	0·00 (0·00)
	1·30 Llolleo	4	24·0	1930	1996	0·65	0·80	0·42
		3	24·0	1870	1996	1·13	1·33	0·02
		2-A	18·3	1610	887			
		2-B	21·3	1740	858			
		1-A	22·6	1810	2340	1·21 (1·38)	1·43 (1·60)	0·02 (0·00)
		1-B	31·9	2090	2463	1·40 (1·63)	1·66 (1·93)	0·03 (0·00)

Table II. *(continued)*

Structure	Ground motion	Level of analysis-variant	D_t (cm)	V (kN)	E_H (kN m)	DM		
						Global	First storey	Top storey
		4	13·7	2390	857	2·47	2·77	0·02
		3	13·7	2440	857	2·81	2·93	0·00
	1·50 El Centro	2-A	14·1	2440	811			
		2-B	15·0	2510	746			
		1-A	14·6	2480	734	2·71 (3·12)	2·83 (3·12)	0·00 (0·00)
		1-B	17·9	2730	793	3·16 (3·74)	3·30 (3·76)	0·00 (0·00)
		4	17·2	2600	659	2·49	2·76	0·02
		3	17·2	2700	659	2·86	2·99	0·00
Frame-Model 2	0·72 Bar	2-A	18·6	2780	647			
		2-B	20·1	2890	605			
		1-A	12·9	2360	648	2·38 (2·71)	2·48 (2·71)	0·00 (0·00)
		1-B	15·3	2540	585	2·54 (3·03)	2·66 (3·04)	0·00 (0·00)
		4	20·6	2850	1925	3·84	4·76	0·07
		3	20·6	2960	1925	5·19	5·47	0·00
	1·30 Llolleo	2-A	21·2	2970	1647			
		2-B	21·8	3020	1573			
		1-A	21·6	3000	3009	6·89 (7·58)	7·33 (7·73)	0·00 (0·00)
		1-B	26·4	3360	3166	7·17 (8·14)	7·88 (8·57)	0·01 (0·00)

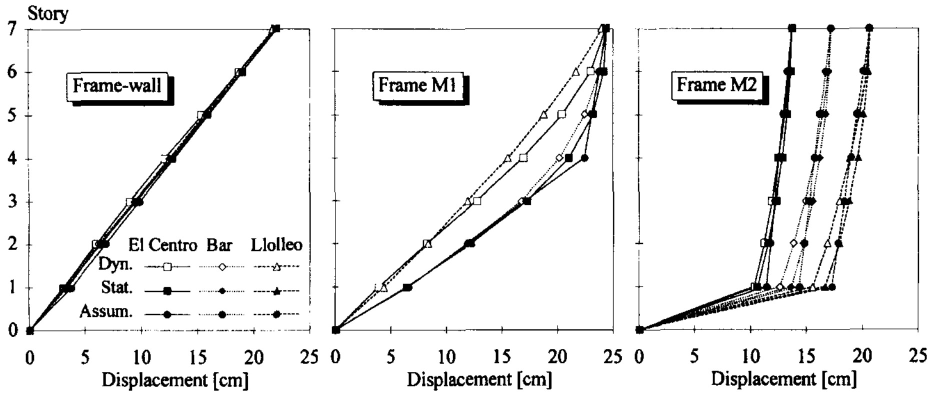

Figure 5. Displacements obtained by dynamic (Level 4 envelopes) and push-over (Level 3) analysis and assumed displacement shapes normalized to maximum static top displacements

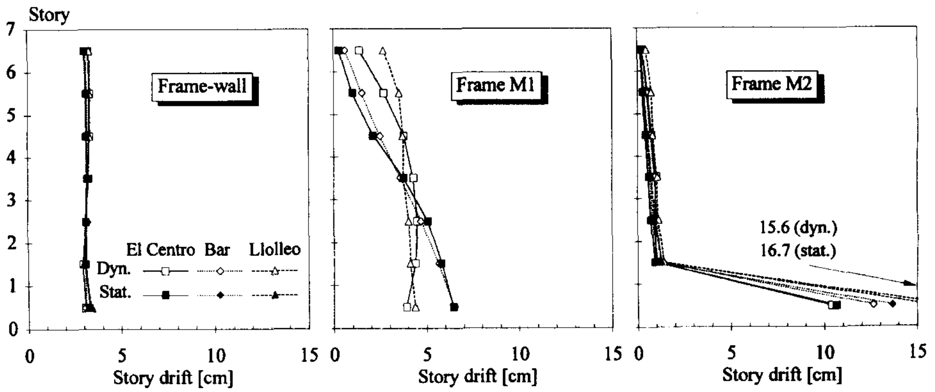

Figure 6. Storey drifts obtained by dynamic (Level 4 envelopes) and push-over (Level 3) analysis

shown). The distributions of displacements and drifts obtained by Level 2 and Level 1 analyses are identical to the distributions computed by Level 3 analysis (the absolute values, however, differ according to the value of the top displacement), and are not shown here. The average rotations, hysteretic energies and damage indices in the beams and columns, as well as in the wall, are shown for each storey in Figures 7–9. Again, the results of the Level 1 and Level 2 analyses are not shown because the distributions along the height are all the same as in the case of Level 3 analysis. The absolute values, however, are not the same, since they depend on the displacement and hysteretic energy demand (the latter can be expressed by the parameter γ). It can be seen that at the local level the differences between the approximate and the 'exact' results are generally larger than at the global level. Better correlation can be observed in the case of rotations than in the case of hysteretic energy. The major discrepancy between the dynamic distribution (Level 4) and the static distributions (Levels 3, 2 and 1) can be again observed in the case of Llolleo ground motion. In frame buildings, demand in the upper part of the structure may be underestimated by the push-over analysis. In a frame-wall building, demand in the lower part of the wall (a few storeys above the basement) may be underestimated as well. In the majority of other cases, however, the accuracy may be considered as adequate for practical purposes.

The effect of the modification of the Park–Ang damage model is small when compared to large uncertainties involved in the seismic capacity of structural members and considering the roughness of the damage model.

The influence of moderate changes in the assumed lateral load distribution along the height of the building has been found to be small. The results are not shown in this paper. However, some ideas about the influence of these changes can be obtained by comparing the results presented here with those given in previous publications.[9,11]

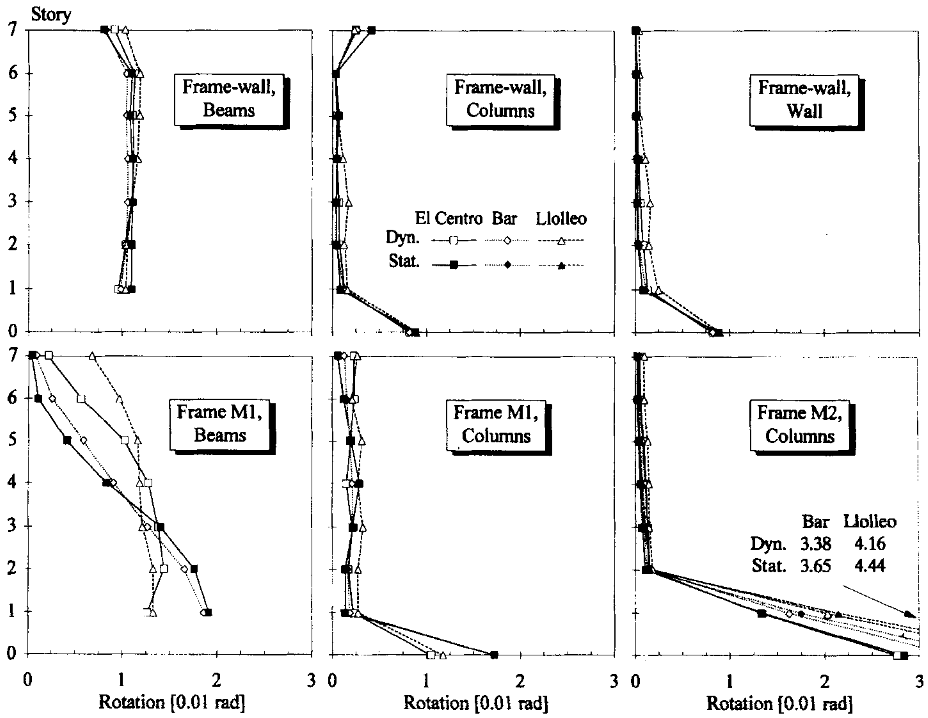

Figure 7. Rotations (mean values for structural members in each storey) obtained by dynamic (Level 4 envelopes) and push-over (Level 3) analysis

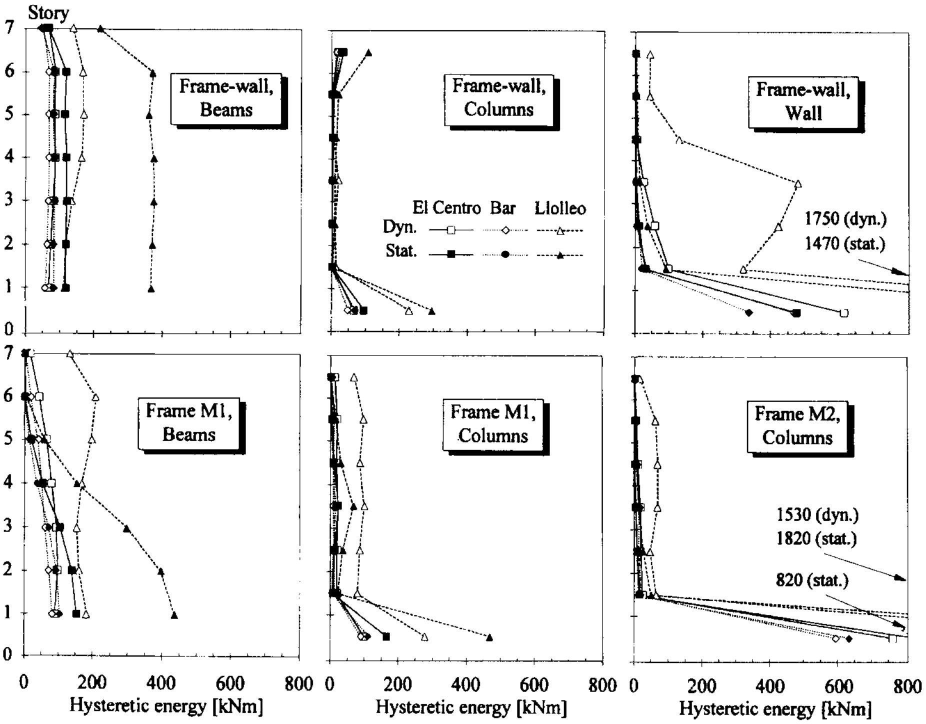

Figure 8. Hysteretic energies (mean values for structural members in each storey) obtained by dynamic (Level 4 envelopes) and push-over (Level 3) analysis

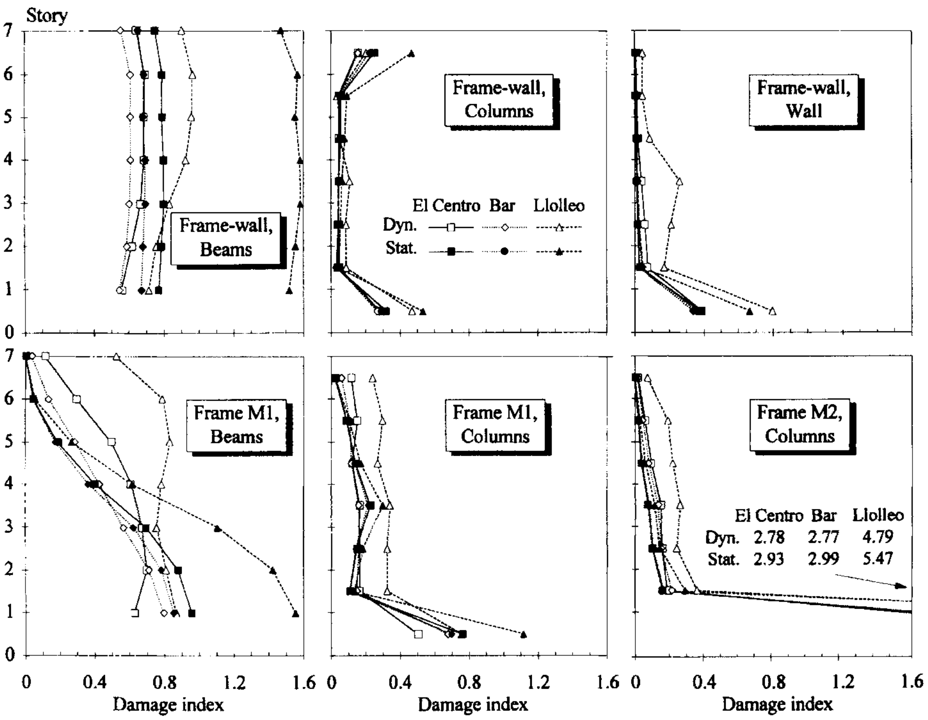

Figure 9. Damage indices (mean values for structural members in each storey) obtained by dynamic (Level 4 envelopes) and push-over (Level 3) analysis

CONCLUSIONS

A simplified method for the non-linear seismic damage analysis of planar building structures, referred to as the N2 method, has been elaborated. It can be applied to the evaluation of both existing and newly designed building structures (in the case of the latter, as the second step of the complete design procedure). For structures that vibrate primarily in the fundamental mode, the method will very likely provide reasonably reliable estimates of global seismic demand. In the majority of cases, the predictions at the local level (demand in terms of deformation and dissipated energy, and damage indices) will be adequate for practical purposes. The method can detect design weaknesses such as a storey mechanism or excessive deformation demands. The influence of different ground motions and structural parameters on the structural response can be easily studied.

If higher mode effects are important, some demand quantities determined by the N2 method may be underestimated. A practical solution to these problems, which has still to be elaborated, might be the appropriate dynamic magnification of selected design quantities.

The results of the N2 method are not excessively sensitive to moderate changes in the assumed displacement shape and the corresponding vertical distribution of lateral loads, as well as in the details of the bilinear force–displacement idealization. Smaller initial stiffness generally leads to larger displacements. The largest uncertainty is introduced by assuming the characteristics of the expected ground motion. However, this is an inherited feature of the problem and usually cannot be avoided in earthquake-resistant design.

ACKNOWLEDGEMENTS

The results presented in this paper are based on work supported by the Ministry of Science and Technology of the Republic of Slovenia. This support is gratefully acknowledged. The authors are indebted to Professor M. Fischinger for important contributions in the initial stage of development of the method, and to Professor H. Krawinkler from Stanford University for stimulating discussions. The authors also welcomed the valuable suggestions of the reviewers.

APPENDIX I

The application of inelastic spectra

Only fundamental relations are given here. For details see References 12–14 and 40.

(a) Input data

Ground motion data: an elastic (pseudo)-acceleration spectrum A_e, and either the estimated value of $\int \ddot{u}_g^2(t)\,dt$ or of the duration t_D. In principle, any smooth elastic spectrum may be used. However, the most convenient is a spectrum of the Newmark-Hall type.

Data on the structural system: mass (m), period (T), damping (ξ), and either yield strength (F_y) corresponding to the bilinear force–displacement relationship or maximum ductility ($\mu = D/D_y$, where D is the maximum displacement and D_y is the yield displacement).

(b) The relationship between strength and ductility:

$$F_y = \frac{m A_e}{R_\mu} \tag{1}$$

Bilinear R_μ-spectra were proposed[12,13] as a function of ductility μ, the characteristic period of ground motion T_C, damping and the hysteretic model.

(c) Maximum displacement:

$$D = \mu D_y = \frac{\mu A_e}{R_\mu \omega^2} \tag{2}$$

(d) Hysteretic energy:

$$\frac{E_H}{m} = (\gamma \omega D)^2 \tag{3}$$

Approximate spectra for the non-dimensional parameter γ[41] were proposed[14] as a function of ground motion (the values of a_g, v_g and $\int \ddot{u}_g^2(t)\,dt$ are needed), ductility, damping model and hysteretic model. In Reference 40 an alternative formula was suggested, where $\int \ddot{u}_g^2(t)\,dt$ was replaced by t_D.

All spectra are consistent. One can start either from strength, or from ductility, or from displacement or even from input energy (not shown here, see References 12 and 14). Different design approaches, such as the displacement-based design approach or the energy-based design approach, can therefore be easily accommodated.

In preliminary designs, typically a value of the target ductility μ or of the maximum acceptable displacement D is assumed, and the required strength F_y is determined using equations (1) and (2) and the formulae for R_μ-spectra.[12,13] In an evaluation procedure, typically the actual strength F_y is known, and ductility demand μ is determined from equation (1) and the formulae for R_μ-spectra. It should be noted that in each case F_y represents the actual strength (the overstrength is included) and not the 'design strength' corresponding to codes. As far as the ductility μ is concerned, reduced ductility, which takes into account cumulative damage (low-cycle fatigue), should be used.[41]

APPENDIX II

The relations between MDOF and equivalent SDOF systems

Several variants have been proposed for the transformation of a planar MDOF system to an equivalent SDOF system (e.g. References 9, 16–19, 42, 43). The variant of the transformation used in this study was derived by assuming a time-independent displacement shape $\{\Phi\}$ (normalized to the top displacement $\Phi_n = 1$) and a vertical distribution of lateral resistance $\{P\} = [M]\{\Phi\}$, where $[M]$ is a diagonal mass matrix. Furthermore, it was required that the original response spectra could be used (i.e. that the transformation of the spectral amplitudes is not needed). The resulting transformation can be written in the form

$$R^* = c\,R \tag{4}$$

where R^* represents the quantities in the equivalent SDOF system (force F^*, displacement D^* and hysteretic energy E_H^*) and R represents the corresponding quantities in the MDOF system (base shear V, top displacement D_t and hysteretic energy E_H) (see Figure 1). Constant c is defined as

$$c = \frac{\{\Phi\}^T[M]\{\Phi\}}{\{\Phi\}^T[M]\{1\}} = \frac{\sum m_i \Phi_i^2}{\sum m_i \Phi_i} \tag{5}$$

where m_i is the concentrated mass at the ith storey. The value in the denominator represents the mass of the equivalent SDOF system ($m^* = \sum m_i \Phi_i$).

Numerical experiments have shown that the results are relatively insensitive to small or moderate changes in $\{\Phi\}$. Consequently, a rough estimate of deformation shape $\{\Phi\}$ can be made. The suggested deformation shapes of three typical systems are shown in Figure 10. For the push-over analysis the lateral load distribution $\{P\} = [M]\{\Phi\}$ should be used.

APPENDIX III

The modified Park–Ang damage model

According to the Park–Ang damage model[15] the damage index DM for each structural member expressed in terms of rotations is defined as

$$\mathrm{DM} = \frac{\Theta}{\Theta_u} + \beta \frac{E_H}{M_y \Theta_u} \tag{6}$$

where Θ and Θ_u are the actual and ultimate rotations, respectively, E_H is the dissipated hysteretic energy, M_y is the yield moment, and β is an empirical constant which depends on the structural characteristics. Θ and E_H represent demand, and M_y, Θ_u and β represent capacity.

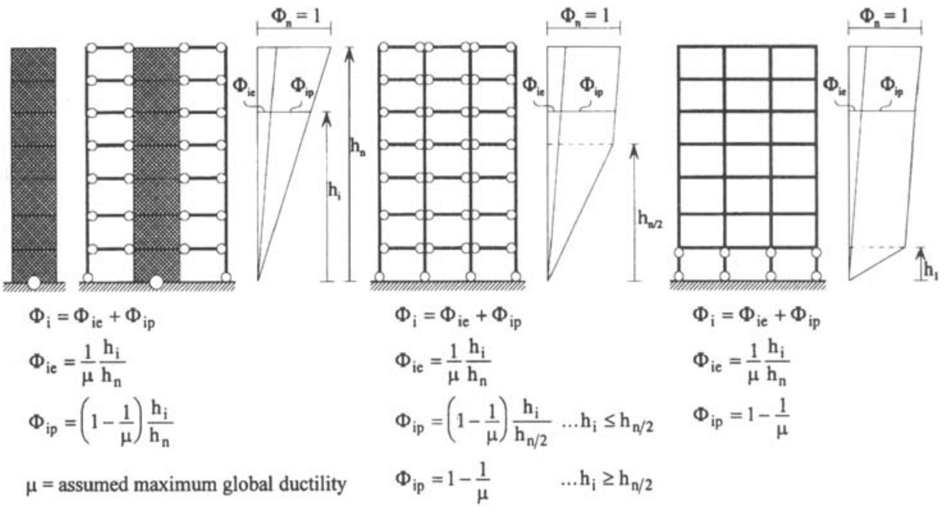

Figure 10. Assumed displacement shapes for different types of building structures

It should be noted that in the original IDARC program,[22] a special interpretation of the E_H in equation (6) was introduced. According to this interpretation, E_H, used in the damage model, is approximately equal to the total hysteretic energy minus the energy dissipated (inelastic part) and absorbed (elastic part) in the case of monotonic loading up to the maximum rotation. The same interpretation is applied in the modified IDARC-L program and all damage indices computed by the original Park–Ang model correspond to this interpretation. In the majority of applications, however, the total hysteretic energy is taken into account in the second term of the Park–Ang model. This more common and conservative interpretation is used in the derivation of the modified damage model given in this appendix.

Let us assume an ideal elastoplastic moment–rotation diagram. In the case of static loading, the dissipated energy, referred to as 'static hysteretic energy', E_{HS}, is defined as

$$E_{HS} = M_y (\Theta - \Theta_y) \tag{7}$$

where Θ_y is the yield rotation. Using equation (7), equation (6) can be transformed to

$$\mathrm{DM} = \frac{\Theta}{\Theta_u}\left(1 + \beta \frac{E_H}{E_{HS}} \frac{\Theta - \Theta_y}{\Theta}\right) \tag{8}$$

In the N2 method, it is assumed that the distribution of dissipated hysteretic energy in dynamic analysis is, throughout the structure, the same as in static analysis. Consequently, the ratio E_H/E_{HS} is constant for all structural members and can be expressed as a function of the parameters of the equivalent SDOF system. In a SDOF system with an ideal elastoplastic force–displacement relationship the static hysteretic energy is defined as

$$E_{HS} = F_y (D - D_y) \tag{9}$$

The hysteretic energy E_H dissipated in a SDOF system subjected to ground motion can be computed as[14,41]

$$E_H = F_y D_y \gamma^2 \mu^2 \tag{10}$$

where μ is the displacement ductility and γ is a non-dimensional parameter (see also equation (3)). From equations (9) and (10) it follows that

$$\frac{E_H}{E_{HS}} = \gamma^2 \frac{\mu^2}{\mu - 1} \tag{11}$$

and the damage index in each structural member can be computed as

$$\mathrm{DM} = \frac{\Theta}{\Theta_u}\left(1 + \beta\gamma^2 \frac{\mu^2}{\mu - 1}\frac{\Theta - \Theta_y}{\Theta}\right) = \frac{\mu_\Theta}{\mu_{u\Theta}}\left(1 + \beta\gamma^2 \frac{\mu^2}{\mu - 1}\frac{\mu_\Theta - 1}{\mu_\Theta}\right) \tag{12}$$

where μ and γ are global values corresponding to the equivalent SDOF system, whereas β, Θ, Θ_u, and the related ductilities μ_Θ and $\mu_{u\Theta}$, correspond to the structural member under consideration.

For determination of the damage index for each structural member according to equation (12) the following quantities are needed: the yield, actual (demand) and ultimate (capacity) rotation, the parameter β, as well as the global displacement ductility μ and the parameter γ. The hysteretic energy is not needed explicitly. However, E_H is included implicitly in γ.

Equation (12) can be written in the form

$$\frac{\mu_\Theta}{\mu_{u\Theta}} = \frac{\mathrm{DM}}{(1 + \beta\gamma^2 (\mu^2/(\mu - 1))(\mu_\Theta - 1)/\mu_\Theta)} \tag{12a}$$

where μ_Θ can be interpreted as an equivalent (reduced) ductility factor for a structural member in a MDOF system (note the analogy with the equivalent ductility factor for SDOF systems[41]). It represents the reduced ductility capacity due to cyclic loading, and takes into account, explicitly, the acceptable damage index DM.

The global damage index can be computed as the weighted average of the damage indices for individual structural members. The weights are assumed to be proportional to the dissipated energy which, according to the assumptions used in the N2 method, is proportional to the static hysteretic energy. Consequently, the global damage index can be defined as

$$\mathrm{DM} = \sum_i \frac{M_{yi}(\Theta_i - \Theta_{yi})}{\sum_i M_{yi}(\Theta_i - \Theta_{yi})}\mathrm{DM}_i \tag{13}$$

where the index 'i' stands for the ith structural member.

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